San Diego Math League Middle School Division, Round 1a
December 8, 2007

1. Which of the following is a prime number?
   (A) 91 (B) 221 (C) 223 (D) 343 (E) 143

2. I have three collars for my dog, one blue, one green, and one red. I have 5 leashes for my dog, one each in green, red, purple, orange, and black. In how many ways can I put a collar on my dog, then put him on a leash so that the leash and the collar are different colors?
   (A) 10 (B) 9 (C) 15 (D) 12 (E) 13

3. Joan is thinking of a positive two-digit number. The number is divisible by 7, and the tens digit is 3 more than the units digit. What is the sum of the digits of Joan's number?
   (A) 12 (B) 11 (C) 10 (D) 9 (E) 8

4. If 6 bits is 75 cents, how many bits equals 4 dollars?
   (A) 32 (B) 16 (C) 8 (D) 36 (E) 24

5. Suppose $x = -\frac{2}{3}$, $y = \frac{3}{4}$, and $z = -\frac{5}{6}$. Order x, y, and z from least to greatest.
   (A) x, y, z (B) x, z, y (C) z, y, x (D) z, x, y (E) y, x, z

6. Evaluate $1 - 3 + 5 - 7 + 9 - 11 + 13 - 15 + \cdots + 33 - 35$.
   (A) -18 (B) 36 (C) -36 (D) 18 (E) -9

7. George likes to draw triangles and squares. During class, he draws triangles and squares all over his paper. No two figures intersect, and no two figures share a side. After drawing all his triangles and squares, he counts how many sides total all his figures have together. He counts a total of 58 sides. He then counts the figures, and finds that there are 16. How many triangles are there?
   (A) 16 (B) 18 (C) 12 (D) 10 (E) 6

8. If $x^2 = 3$ and $y^3 = 2$, what is $(xy^2)^6$?
   (A) 216 (B) 432 (C) 648 (D) 1728 (E) 512

9. ABC is a right triangle with $\angle ABC = 90°$. Find the area of $\triangle ABC$ if $AC = 25$ and $BC = 24$.
   (A) 150 (B) 84 (C) 300 (D) 120 (E) 144

10. Given that
$$\left(\cdots \left(\left(17^{\frac{2}{3}} \right)^{\frac{3}{4}} \right)^{\frac{4}{5}^{\cdots}} \right)^{\frac{99}{100}} = \sqrt[100]{x}$$
    What is x?
    (A) $\sqrt{17}$ (B) $\frac{17}{2}$ (C) 17 (D) 34 (E) 289

11. A caveman is making 5 piles of stones by placing one stone at a time on a randomly chosen pile. What is the smallest number of stones he must use in order to guarantee that some pile contains at least 10 stones?

    (A) 10 (B) 45 (C) 46 (D) 49 (E) 51

12. Ranger the dog is tethered to a pole with an 10 ft long rope at the corner of a building that is shaped like a regular pentagon (when viewed from above). The sides of the building are each 20 ft long, and Ranger is outside the building. How much area, in square feet, can Ranger reach?

    (A) 14π (B) 30π (C) 70π (D) 100π (E) 280π

13. The sequence $0, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 4, ...$ is formed by writing each nonnegative integer n a total of 2^n times, in increasing order. What is the 2007th element of the sequence?

    (A) 9 (B) 10 (C) 11 (D) 512 (E) 1024

14. Ben is sharpening pencils, which are initially cylinders of diameter 1 cm, with a pencil sharpener that can hold 10 cm^3 of pencil shavings. He sharpens each so that the total length of the pencil remains the same, but the tip is a cone with height 2 cm. How many whole pencils can he sharpen before having to empty the sharpener?

    (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

15. A spherical container of radius 25 is partially filled with water. The depth (measured perpendicular to the surface) at the deepest part of the water is 18. What is the area of the top surface of the water (the water not touching the sphere)?

    (A) 576π (B) 625π (C) 49π (D) 324π (E) 441π

San Diego Math League Middle School Division, Round 1b
December 8, 2007

1. Find x if $\dfrac{1+\frac{3}{x}}{2-\frac{2}{x}} = 7$.

2. How many positive divisors does 200 have?

3. If n is a positive integer such that $1+2+3+\cdots+n = 190$, then what is n?

4. The Flyfishing Club is choosing officers. There are 23 members of the club. 14 of them are boys and 9 are girls. In how many ways can they choose a President and a Vice President if one of them must be a boy and the other must be a girl (either office can be held by the boy or the girl)?

5. Maria and Joe are jogging towards each other on a long straight path. Joe is running at 10 mph and Maria at 8 mph. When they are 3 miles apart, a fly begins to fly back and forth between them at a constant rate of 15 mph, turning around instantaneously whenever it reaches one of the runners. How far, in miles, will the fly have traveled when Joe and Maria pass each other?

6. Find the smallest positive integer k such that $k!$ ends in at least 43 zeroes.

7. Each of the first 150 positive integers is painted on a different marble, and the 150 marbles are placed in a bag. If n marbles are chosen (without replacement) from the bag, what is the smallest value of n such that we are guaranteed to choose three marbles with consecutive numbers?

8. A square sheet of paper that measures 18 cm on a side has corners labeled A, B, C, and D in clockwise order. Point B is folded over to a point E on $\overline{AD}$ with $DE = 6$ cm and the paper is creased. When the paper is unfolded, the crease intersects side $\overline{AB}$ at F. Find the number of centimeters in FB.